AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

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No. 860.

PILE-DRIVING FORMULAS: THEIR CONSTRUCTION AND FACTORS OF SAFETY.

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Presented September 6th, 1899.

WITH DISCUSSION.

Supporting Piles.

Experiments on the impact of the ram of a pile-driver, from the ordinary heights of its operation, and deductions therefrom, are not as satisfactory as the importance of the subject requires, as evidenced by the variety in the elements of computation, formulas and factors of safe loads.

Authors upon physical laws, and engineers who have essayed to give a formula for the impact and effect of a ram on a pile, vary not very materially, but in some instances have introduced elements having little or no connection with it, and consequently of no essential value: as the diameter, weight and length of a pile, its mean depression per number of blows, its modulus of elasticity, and the cube root of the height of fall of the ram.

The resistance a pile offers to the blow of a ram is the measure of

its capacity to sustain a stress, wholly independent of its diameter, weight, length, modulus of elasticity, etc.

Thus, the diameter of a pile and the distance of its penetration are not proper elements in the computation, as their effect is embraced in their limitation of the penetration of the pile; added to which the weight of the pile is of consideration only as involving a weight of ram proportionate to its inertia, and the relative elasticity of a pile is of little moment, for when a pile approaches to the penetration of required resistance or refusal, if its head is broomed it is dressed off, and if split or liable to be, it is confined by a ring. In fact, the weight and fall of a ram being proportionate to a pile, all other elements, except that of penetration, are rendered of no consideration, as elements of the computation of effect; and particularly when so great a factor of safety as the varied conditions under which a pile is driven requires, and which is given by various authors of formulas as ranging from 0.083 to 0.33, omitting Weisbach's extreme of $\frac{1}{100}$.

In the following treatment of the subject, a pile is considered as driven to a final depression of 0.5 in. or less, with a level and sound head, as far as the operation renders it practicable; and when the weight of the superstructure to be sustained requires the number of piles to be increased proportionately thereto, this increase objectionably involves the necessity of placing them at less than a proper distance apart, for when they are driven close in a moist or clayey soil, the last one driven will, by the lateral compression, cause the adjoining ones to rise.

In the formulas of Major Saunders, U. S. A., Molesworth, Weisbach, Mason, Trautwine, Nystrom, Wellington and the "Dutch," the final depression or penetration, either in inches, feet or *pro rata*, is given as a divisor. Hence, assuming that the impact of a ram in a given case is 100 000 lbs., and the depression is 0.5 in., 100 000 ÷ 0.5 = 200 000 lbs., and if it is 0.25 in., 100 000 ÷ 0.25 = 400 000 lbs.; and in like manner to infinity or positive refusal; and yet the impact would not have increased, as the effect of the blows is not cumulative. The resistance of a pile is not measured by a summation of the several impacts it has received by the fall of the ram, or by the *pro rata* of its depression per number of blows, as by the "Dutch" formula. When a street paver depresses a stone 1 in. with his rammer, and 0.5 in. at a succeeding blow, he has not correspondingly increased the force or

effect of it, neither are the blows of a carpenter in driving a bolt cumulative, for only the last blow gives the measure of the impact on it. Hence, the effect of the blow of a ram, however apparent to observation, is lost to consideration when the one that follows it occurs, as the measure of the last is the measure of the resistance the pile offers, and that resistance is the single sustaining stress or weight which the pile will bear.

The matter, then, is not how the resistance of a pile is obtained, not what the weight of it, not what its inertia or the effect of its cushioning, but what was the power required to drive it to its final depression, or what is the measure of its resistance to stress or to sustaining a weight?

Illustration of the Several Formulas Referred To.

Assume rams having weights of 1 000 and of 2 000 lbs., falling, respectively, 20 and 25 ft., which are taken as exponents of ordinary practice. Mean section of pile, 100 sq. ins.; weight, 1 000 lbs.; surface, 130 sq. ft.; length, 35 ft.; and penetration under last blow, 0.5 in. = 0.043 ft.

(1) *Major Saunders.* $\dfrac{R\,h \div d}{8} = W.$

R representing weight of ram and W safe load, both in pounds; $h =$ height of fall, and $d =$ depression of last blow, both in feet.

(2) *Rankine.* $\sqrt{\left(\dfrac{4\,E\,s\,W\,h}{l}\right) + \dfrac{4\,E^2\,s^3\,d^2}{l^2}} - \dfrac{2\,E\,s\,d}{l} = L.$

$E =$ modulus of elasticity, $s =$ section of pile in square inches, $l =$ length of pile in feet, and $L =$ dead load without further depression. Other symbols as preceding. Factors of safety, 5 to 1.

This formula embraces modulus of elasticity, length and sectional area of piles, all of which, for the causes here given, are held to be an unnecessary complication and of no practical value.

A Second Formula by Rankine. $A\,1\,000 = L$ at refusal of pile; and when not, $A\,200 = L$; or a factor of safety of 5.

$A =$ mean area of section of pile in square inches, and $L =$ load in pounds.

(3) *Molesworth.* $\dfrac{W\,H}{8\,D} = L.$

$W =$ weight of ram, and $L =$ safe load, both in hundredweights. $H =$ height of fall, and $D =$ set of pile by last blow, both in inches.

(4) *Mason.* $\dfrac{W^2\,h}{S(W+p)}$ $=$ extreme load.

$S =$ depression by last blow, in feet; $p =$ weight of pile in pounds; other symbols as preceding. Factors of safety, 10 to 4.

(5) *Trautwine.* $\dfrac{\sqrt[3]{h\,W\,0.023}}{s+1} \times 2\,240 =$ extreme load. $s =$ depression by last blow in inches; other symbols as preceding. Factors of safety, 10 to 2.

(6) *Nystrom.* $\dfrac{M^3\,S}{6\,s\,(M+m)^2} =$ safe load.

$M =$ weight of ram, and $m =$ weight of pile, both in pounds. $S =$ fall of ram and s depression, both in inches; other symbols as preceding.

(7) *Wellington.* $\dfrac{2\,Wh}{s+1} =$ extreme load.

$s =$ depression by last blow, in feet; other symbols as preceding.

(8) *McAlpine.* $80\,(W+0.228\sqrt{F}-1) = X.$

$W =$ weight of ram, in tons; $F =$ fall of ram, in feet, and $X =$ load at refusal. Factor of safety, 0.33.

TABLE No. 1.—RECAPITULATION OF RESULTS. DEPRESSION IN ALL CASES, 0.5 IN.

Formulas.	Factors, as given by the several Authors.	1 000 LBS. FALLING 20 FT.		2 000 LBS. FALLING 25 FT.	
		Load.	Safe load.	Load.	Safe load.
		Pounds.	Pounds.	Pounds.	Pounds.
1. *Saunders*........	8	476 000	59 500	1 190 000	148 750
2. *Rankine*.......⎰	5 to 1	332 125	66 425	646 246	129 249
⎱	5	100 000	20 000	200 000	40 000
3. *Molesworth*......	8	480 080	60 010	1 199 520	149 940
4. *Mason*...........	10 to 4	273 598	⎰ 27 360 ⎱ 68 399 ⎰	861 326	⎰ 86 133 ⎱ 21 533 ⎰
5. *Trautwine*	12 to 2	93 218	⎰ 7 768 ⎱ 46 609 ⎰	200 592	⎰ 16 716 ⎱ 100 296 ⎰
6. *Nystrom*.........	6	166 086	27 681	658 434	109 739
7. *Wellington*......	....	....	38 387	....	95 970
8. *McAlpine*........	3	83 328	27 776	184 934	61 644
Mean of results.....	5.75	250 554	40 901	645 131	87 271

* A mean by the author.

The varying figures in the fourth and sixth columns, under safe load, represent results as computed by the formulas of different authors, with their factors.

The following formulas were not included in the preceding reca-
pitulation and summation of results, as the factors in the first are held
to be too extreme for practical application, and the introduction in
the other, of the average penetration per blow, per 100 blows of the
ram, is not an element of any value; for, as before stated, the final
blow of a ram is the only one which measures its effective value.

Weisbach.—One of Weisbach's formulas embraces elements similar
to those of Rankine, with a factor of safety of 10 to 100. A second

$$\frac{w^2 h}{s(w+p)} + (w+p) = \text{extreme load.}$$

$w =$ weight of ram, and $p =$ weight of pile, both in lbs.; $s =$ pene-
tration by last blow, in feet $= 0.043$; other symbols as in the other
formulas. Factors of safety, 100 to 10.

" *Dutch.*" $\dfrac{B^2 h}{(B+M)e} = W.$

$B =$ weight of ram, and $M =$ weight of pile, both in lbs. $e =$
average penetration per blow for 100 blows, and $h =$ height of fall,
both in inches. Factor of safety, 6.

In the "Dutch" formula, e represents the average depression
of the pile per blow, per 100 blows of the ram, which is not only
an element measurably impracticable of attainment in practice, but
is wholly valueless and difficult of attainment except with the heavy
ram and low fall practiced by the authors of the formula.

A recorded case furnishes the following illustration of the formula:
B and M, respectively, 2 205 and 2 030 lbs. and $e = 0.28$ in.

$$\frac{2\ 205^2 \times 20}{(2\ 205 + 2\ 030)\ 0.28} \div 6 = 13\ 665 \text{ lbs.}$$

Operating this formula with the elements given in the first of the
preceding cases, the result would be 35 715 lbs.

Factor of Safety.—In deciding upon the factor of safety the follow-
ing elements are to be considered:

The friction of the guides of the leader and of the hoisting line of
the ram in the sheave and over the drum (ascertained by experiment
with a very heavy ram to be 0.2 ft. of penetration, but with a light
ram it would be materially more); the resistance of the atmosphere to
the fall of the ram and its cushioning on the head of a pile however
squarely it may be dressed off; the want of verticality, both of the fall
of the ram, and of the vertical plane of the pile and its consequent

lateral vibration; the inertia and frequent splitting of the pile on a boulder; the vibration, if driven in a trench or with a "follower"; the condition of the soil, whether wet, moist, or dry, that is, if it is wholly dry soil, if the pile is embedded or partially exposed above ground or water, or is wholly immersed in wet soil or surface water, all of which are elements in the determination of the factor.

In this connection is to be considered the subsidence of the soil around a pile which has been temporarily arrested in its driving, the effect of which, in the course of a few hours under favorable conditions of soil, has been observed to approach to that of the resistance of a pile at its last blow.

The following are records of some ascertained sustained weights on piles:

1. At the London Bridge over the Thames,* the piles are reported to sustain 80 tons. Assuming them to have been driven under the extreme condition of a ram of 1 500 lbs. falling 20 ft., their sustaining power in excess of that given by the formula $W \sqrt{2\,g\,h}$ would be 23.94 tons, and $80 \div 23.94 = 3.33$ to 1.

2. A pile driven with a ram of 2 000 lbs. falling 5 ft. supported 47 tons, an excess over the formula of $16 \sim 47 = 31$ tons; and $47 \div 16 = 2.93$ to 1.

3. A pile driven with a ram of 1 700 lbs. falling 16 ft. supported 70 tons, or an excess of $24.3 \sim 70 = 48.7$ tons, or as 2.88 to 1.

4. A test pile by the United States Government, driven by a ram of 910 lbs. falling 5 ft. with a depression of 0.375 in. under the final blow, supported 26.6 tons, or an excess of $7.28 \sim 26.6 = 19.32$ tons, or 3.65 to 1.

The average of results for safe loads, deduced by the several formulas given are, for a ram of 1 000 lbs. falling as given, 40 901 lbs.; a result 2.04 times greater than that given by the formula of an author of highest recognized ability, and 1.46 times less than by another author of similar standing. For a ram of 2 000 lbs. falling as given, the average is 87 271 lbs.; a result 2.182 times greater than that given by the first author referred to, and 1.35 times greater than that given by the other.

In view of this variance, the author essayed to ascertain the difference between the results by the formula, or foot-pounds, and the actual

* "Trautwine's Engineer's Pocket-Book," pp. 643-644.

vis viva of a falling body; and although the experiments were necessarily on a very limited scale, the following results were obtained:

Weight.	Fall.	Velocity.	Impact.	Ratio of Impact to Velocity.
Pounds.	Feet.	Feet.	Pounds.	
1....................	1	8	32	4.0
1....................	1.5	9.5	42	4.3
1....................	2	11.31	52	4.6

Reviewing these and the various other elements submitted, it appears that all the formulas here illustrated, for piles driven under the usual requirements of practice, as to weight of ram, height of fall, character of soil and final depression, are but arbitrary, with factors of safety having the wide range of three-tenths to one-twelfth. They present even the varied conditions of the weight multiplied by the height, by its square, and also by its cube, while that of Wellington presents the exceptional condition of the weight being multiplied by twice the height of the fall; all of them being at variance with the accepted rule, that the velocity of the weight and its resultant impact is as $\sqrt{2\,g\,h}$. Hence, for heights of fall of 16, 25 and 36 ft., the product of the Wellington formula would be as 32, 50 and 72 ft., while the velocity of impact would be as 32, 40 and 48 ft.

Further, that the impact or *vis viva* of a body falling freely, as determined by the experiments here referred to, exceeds four times that given by the formula for its final velocity, which result is verified by the illustrations given of the actual support of piles noted, to which should be added an estimate of the frictional loss of the ram in operation.

Reviewing the elements presented, the following formula is submitted: $\dfrac{4\,w\sqrt{2\,g\,h}}{F}$, or $\dfrac{4\,w\,8\,\sqrt{h}}{F}$ = safe load in pounds.

w = weight of ram in pounds, and h = height of fall in feet.

F, or the factor of safety, in consideration of the several losses of effect recited, and especially that of the brooming of the head of a pile, and the driving of piles in trenches or with a "follower," is assumed at from 3 to 6, according to the nature and condition of the soil, the character of the piles and the integrity of their driving.

Applying the formula to elements similar to those given for the

several formulas recited, the results would be, for the illustration of a ram of 1 000 lbs. falling 20 ft., with a factor of 3:

$$\frac{4 \times 1\,000 \times 8 \sqrt{20}}{3} = L;$$

and for the ram of 2 000 lbs. falling 25 ft., eliminating the numerator 4, and correspondingly reducing the divisors, 3 to 6, to 0.75 and 1.5, the operation is simplified, thus:

$$\frac{1\,000 \times 8 \sqrt{20}}{0.75} = 47\,701 \text{ lbs., and } \frac{2\,000 \times 8 \sqrt{25}}{0.75} = 106\,666 \text{ lbs.};$$

and with a factor which is a mean between 0.75 and 1.5 = 1.125, the results would be, respectively, 31 801 and 94 814 lbs., results in accordance with the observation of an exceptionally extensive and varied experience in the operation under consideration.

In further support of the formula submitted, the following is presented:

First.—That at London Bridge, the sustaining power of the piles is reported to be 3.33 times the weight deduced by the ordinary formula for the impact of a falling body; in other cases to have been 2.93 and 2.88 times, and in a test case by the United States Government to have been 3.65 times, giving a mean of 3.1975; to all of which is to be added the greater or less losses of effect of the ram in falling, as herewith detailed.

Second.—That the test pile previously cited, which was driven and observed by officers of the United States Government, the ram weighing 910 lbs., the fall being 5 ft., and the depression of the pile at the final blow being 0.375 in., sustained a stress of 26.6 tons; that the sustaining power of the pile, deduced by the ordinary formula for the velocity of a falling body, would be $910 \times \sqrt{2\,g\,h} = 16\,457$ lbs. $= 7.34$ tons; and that these results of 7.34 and 26.6 are as 1 to 3.62, approximating closely to the 4 assumed in the formula presented.

Third.—That piles driven to sustain the vibratory stress of a bridge under a hammer of 1 200 lbs., falling 20 ft., with a penetration of 0.75 in., enduringly sustained 18 tons each, and by the formula the result is 17 tons.

Regarding the distance from centers at which piles should be driven; in sand or small gravel, this may be 2 ft., but for saturated sand and earth, or in silt, a greater distance is required, as small piles are likely to be disturbed in their position by the driving of a larger one adjoin-

ing them, and in some practice, in determining the capacity of a range of piles, it is proper to reduce the result obtained by the formula, to meet incidental conditions, such as negligence in driving, or in super-intendence, the frequent and unobserved splitting or crushing of a pile on a stone or boulder.

The deductions and opinions here given are the result of observations of a very varied and extensive experience in pile-driving, in every variety of soil, encountering quicksand, boulders, submerged crib-work, stone filling, in water, and under the embarrassing conditions met in a trench, when the hammer was necessarily arrested many feet above the designed level of bearing of the piles, and for structures of various capacities and heights, some attaining twelve stories; and whenever practicable, a bench-mark was established and ultimate results observed.

Tenacity of Piles.—The effects of the tenacity were illustrated at Cuxhaven, England, with a fir pile, 1 ft. in diameter at the head, driven 15.7 ft. in fine and drifting dry sand; and which, at the termination of 23 days, required a power of 17 472 lbs. (7.8 tons) to withdraw it; also one of 17 ins. diameter, driven to a like depth, at the termination of 20 years, required a power of 57 120 lbs. (25.5 tons), and at 50 years, 98 000 lbs. (43.75 tons), to remove it.

Friction of Pine Sheet Piles in Clay.—The removal of a coffer-dam after 5 years' existence furnishes the following elements:

Piles of Memel fir, 12.5 ins. square, length 40 ft., driven to an average depth of 18.25 ft., the average superficial area in the soil being 76 sq. ft., the net resistance, after deducting the weight of the piles, etc., was 71 277 lbs., giving a coefficient or factor of 93.8 lbs. per square foot for the bearing of the clay on the inner and outer surface of the piles.

Follower.—Whenever a follower is used, however short it may be, the factor of safety should be increased; and when piles are driven in trenches below the driver, and their final depression below the drop of the ram is so much as to involve the use of a long follower, the extreme factor should be used; and with piles of spruce or like wood, their heads should be ringed as soon as their penetration becomes difficult.

Brooming.—The result of brooming the head of a pile, in conse-quence of its resistance, as determined by experiment, was to increase

the number of blows of a ram compared with a sound head from 1 to 1.49.

Sheet Piling.—Sheet piling would be much more effective, either in water or wet soil, if constructed of two or more thicknesses of timber laid together and lap broken for half the width of a piece if two are used and one-third the width if three are used, and fastened with locust, or iron blunt bolts, the number of which and their distances apart being determined by the stress of the driving. The off edge of the foot of the pile should be cut or tapered to an extent which will tend to lead and retain it close to the adjoining pile.

Notes Deduced from Operations of Various Authors on Piling.—In wet sand, silt, or any light soil, piles should be driven rapidly, otherwise the subsidence of the material in which they are driven will furnish increased resistance.

In strong or stony soil, a heavy ram with a low fall is better than a light ram and a high fall. If the soil is very strong with stone, the piles should be shod.

Inasmuch as a stratum of light or wet earth may exist below a heavy one, before commencing piling upon a new soil, in order to be assured, not only of its consistency, but also of the depth of such condition, a test pile of small diameter should be driven to as great a depth as practicable.

The support of compressed sand is estimated at 85 lbs. per square foot, and the friction on the surface of a pile at 614 lbs. per square foot.

Finally, in a recent case, in consequence of the foundation of a building of eleven stories being 7 ft. below that of two exceptionally heavy adjacent structures of ten stories each, the walls of which were laid on a concrete bed over wet sand, piling was necessarily arrested at distances of 7 ft. from these adjoining walls, and the spaces bridged with plate-iron cantilever beams. The result, as verified by bench-marks, has proved the sufficiency of the piling under the concentrated stress over the voids, which was spaced and driven in accordance with the formula here submitted.

DISCUSSION.

FOSTER CROWELL, M. Am. Soc. C. E.—The author's experiments Mr. Crowell.
upon the impact of falling weights, as recorded in the table on page
273, having been made with a spring-balance, the observed results were
the resistances of the spring-balance, exerted through unrecorded dis-
tances of retardation of the falling weights, and therefore, if the ap-
paratus had happened to be of a different degree of sensitiveness the
values would necessarily have been correspondingly different; conse-
quently the results could not be taken as the absolute values of impact
of the falling weights, to arrive at which it would be necessary to de-
termine accurately, not only the resistances developed, but also the dis-
tances of retardation. The experiments, therefore, could not be ac-
cepted as establishing, even practically, the author's claim, upon which
he bases his formula. For, whatever the weight of the impinging body,
or the velocity of the impact, there is, in every case, a finite amount
of work U, yielded upon the opposing resistances; there is in every
such case a certain mean resistance R, overcome through a certain
space S, in the direction in which that resistance acts, which resistance
and space are such that $R S = U$, and $R = \dfrac{U}{S}$; if, therefore, the space
S be exceedingly small as compared with U, there will be an exceed-
ingly great resistance R, overcome through that small space, and *vice
versa*. Taking as the values of R, the "impacts" of the table, we find
that the corresponding values of S, in inches, are respectively 0.375,
0.410 and 0.461. Thus it is apparent that, had the apparatus been so
constructed as to weigh with less movement, the recorded impact might
have been very much higher; also, that the movement of the apparatus
is analogous to a pile penetration of about 8 ins. under a 20-ft. fall of
the ram, in the first case given, but to only about $5\frac{1}{2}$ ins. in the third
case, which shows the influence of the friction and lost motion in the
apparatus.

Mr. Haswell's formula, as presented, is limited to a penetration
of $\frac{1}{2}$ in., and it would be interesting to know what provision the
author intended to be made in cases where the observed penetra-
tion of the driven pile exceeded that amount, as such cases, in
the speaker's experience, constituted the real test of reliability and
usefulness of a pile-driving formula. The ideal formula, and the
practical one as well, should contain within itself the elements of
application to all the cases usually to be met with in practice, and
on that account, as well as upon others, the speaker considers the
Wellington form of formula preferable to that of Mr. Haswell.

The author has condemned the Wellington formula, including it
with those which are arbitrary and at variance with accepted rules,

Mr. Crowell. and specifically because it "presents the exceptional condition of the weight being multiplied by twice the height of the fall," claiming that the true formula should be founded upon the actual *vis viva* of a body falling freely, as represented by its velocity, viz., $\sqrt{2\,g\,h}$. This is a misconception, both as to the fact and as to the theory of *vis viva*. The *vis viva* of a body falling freely being $M v^2$, and h being the height through which the body must fall to acquire the velocity v, $v^2 = 2\,g\,h$; therefore $h = \dfrac{v^2}{2\,g}$, and $Wh = \frac{1}{2}\,M v^2$; that is to say, the work accumulated in a body falling freely, available for extraneous effect, is expressed by half the *vis viva*, and is always equal to the simple product of its weight multiplied by the distance through which it has fallen. This effect may be expressed in foot-pounds, or inch-pounds, or half-inch-pounds, or by any other combination of weight and fall, but it can never by any possibility exceed itself, and is an immutable feature of the laws of falling bodies applicable to all heights.

Vis viva, according to Mosely,[*]

"Does not represent a pressure, or force, but the numerical equivalent of the product of a certain number of units of pressure and a certain number of units of path. The one magnitude being of as totally different order from the other as an area is different from a line, and therefore having no common unit of measure."

Impact and *vis viva* are thus totally different things. What enables the descending ram to drive the pile is the energy stored therein, but not all of that energy is available for the purpose, half of it being required to maintain the movement of the ram. That which is available is known as the accumulated work, to be expended upon the sum of the resistances offered by the pile.

In the Wellington formula the principle of accumulated work is the basis; it is expressed in inch-pounds, but as it contains a silent factor of safety of 6, and, as, for convenience, h is given in feet, the accumulated work is written $2\,W\,h$ instead of $W\,12\,h$. The denominator is the measure of resistance developed by the pile, as indicated by its penetration under the last blow, or settlement, s, also expressed in inch terms. Were it not for the presence of resistances in the pile-driving machine, and other causes tending to oppose the driving of the pile, s would be the full measure of the net resistance, and the quotient would be that resistance; but these various unmeasured opposing forces must be taken into consideration, and they are represented by the constant quantity, 1, in the denominator.

Thus we have the derivation of the Wellington formula,

$$Q = \text{Safe Load} = \frac{2\,Wh}{s+1}.$$

* "Mosely's Mechanics of Engineering," p. 589.

Comparing this with Mr. Haswell's formula, which is

$$\text{Safe Load} = \frac{8\ W\ \sqrt{h}}{1.125},$$

we perceive that for the $\frac{1}{2}$-in. penetration, to which the latter applies, it becomes $Q = \dfrac{2\ W\ h}{1.5}$, and if we assume a fall of 16 ft. we shall have
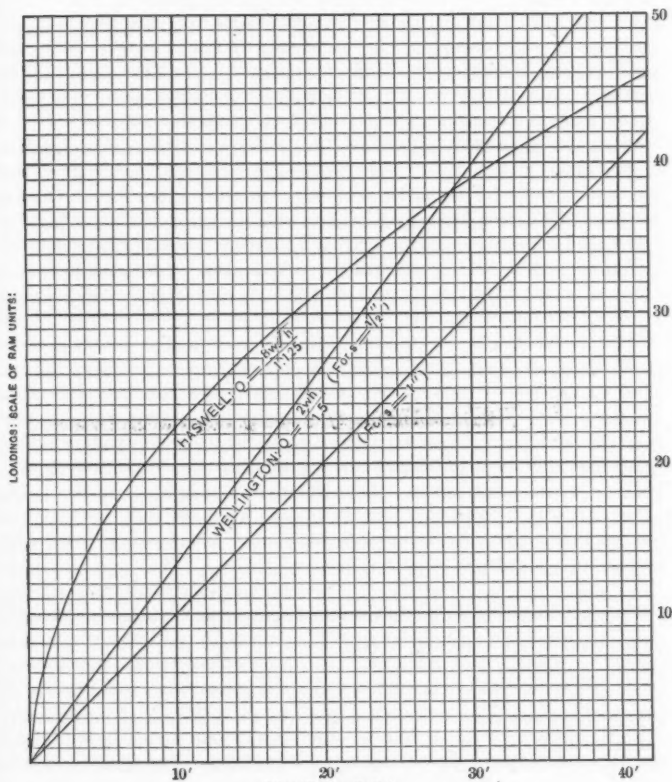

FIG. 1.

by the Wellington, $Q = \dfrac{32\ W}{1.5}$; and by Haswell, $Q = \dfrac{32\ W}{1.125}$; results differing only in the factors of safety. That is to say, with the same actual factor of safety the results for the two formulas would be identical for the assumed fall of 16 ft.

Mr. Crowell.   The assumed factor of safety in the Wellington formula is 6, while the assumed factor in the Haswell is only $4\frac{1}{2}$; actually, the above Wellington expression gives $Q = 21.333\ W$, and the Haswell $Q = 28.444\ W$.

The two will be found to give absolutely the same working result with a fall of 28.45 ft. A study of the diagram, Fig. 1, of the characteristic plottings of the respective formulas may be instructive. The Haswell formula is the equation of a parabola and Wellington's is a straight line; the abscissas are the heights of fall of the ram and the vertical ordinates the safe loads expressed in units of ram weight. It will be noted that for low falls the Haswell formula gives relatively great safe loads and for high falls relatively smaller loads.

Considering that the highest value given by the Wellington line has a theoretical factor of safety of 6, it is apparent that there is no advantage in the drop of the Haswell curve at the higher falls. On the other hand the exceeding great values, relatively, given by it for the low falls show its inconsistency and the danger of its use by the inexperienced. It will be seen that for a fall of only 2 ft., the indicated safe load is one-third as great as that indicated for a fall of 18 ft., nine times as great.

The lower of the two inclined straight lines in the diagram is introduced to show the Wellington values when the penetration is 1 in. It will be observed that the margin of safety is somewhat decreased, so that, strictly, the supplementary quantity in the divisor should not be a constant, but a variable, as was pointed out by the speaker in the discussion on his own paper on this subject, presented before the Society several years ago.* There can be no question, however, that for ordinary penetrations occurring in practice the Wellington formula is amply safe, and, in the speaker's opinion, its simplicity and ready applicability entitle it to preference.

Mr. Gould.   E. SHERMAN GOULD, M. Am. Soc. C. E.—It is needless to say that, in the speaker's opinion, this paper has great practical value. The long experience of the author in the kind of work described, and his mature judgment, give importance to his utterances. He exposes the vast array of irrelevant elements introduced by many authorities into a problem which, from its nature, does not admit of mathematical refinement, and presents a formula which, in the speaker's opinion, has much to recommend it. Its chief peculiarity is in assuming the constant value of $\frac{1}{2}$ in. for the final set, instead of a varying one. That is to say, his formula assumes what amounts practically to "refusal," or the absence of further penetration after the last blow of the hammer.

The problem which confronts us in pile-driving formulas, is to find

* "Uniform Practice in Pile-Driving," *Transactions*, Am. Soc. C. E., Vol. xxvii, pp. 108, 596.

the true relation between the effect of the sudden impact of a falling Mr. Gould.
heavy body, and the steady pressure of a quiescent one. No such
relation has yet been discovered, and all that can be done is to seek an
empirical expression which will be a safe, practical guide. The
formula quoted by the author as "Wellington's," is probably the one
which has most endeared itself to American engineers from its sim-
plicity and generally safe results. Its denominator is a binomial
expression containing the variable set of the pile, or its final penetra-
tion, plus unity. In this form it is relieved of the absurd results
which would otherwise ensue from taking very small values for the
set, which absurdity would reach its climax if we should assume abso-
lute, or zero set, when the formula would give an infinite bearing
capacity to the pile, no matter how light the hammer or how low the
fall. This impossible result is avoided by using $s + 1$ in the denomi-
nator.

Wellington's formula, however, like all others recognizing move-
ment or set in the pile under the last blow, fails to meet the require-
ments of the problem. It is merely the statement of the equality
between the work stored in a heavy body falling through a certain
height, and a force, called the "safe load" moving through a relatively
short distance called the "set." There is no quiescent pressure
involved, nor can there be. Work in the hammer can only be offset
by work in the pile, and work implies movement and not rest. The
"safe load" is not, therefore, according to these formulas, the steady
pressure which the pile could bear at the moment of receiving the last
blow, but the equivalent pressure which would at that moment force
the pile down the distance of the final set.

After receiving the blow it may be assumed, on empirical princi-
ples, that the pile could bear that weight without any further move-
ment.

These considerations appear to the speaker to meet the question:
What formula can be used safely in the case of a set of considerable
amount, 6 ins., for instance ? The answer appears to him to be:
None at all. He considers that, in order to use any formula success-
fully, the piles, in all cases, should be driven to a final set of not more
than 1 in. If a pile 40 or 50 ft. long continues to go down as much as
6 ins. under the hammer, then, to apply a formula. the weight or fall
should be reduced until only 1 in. set is produced, and its bearing
capacity then calculated by whatever formula may be preferred. If the
final set is taken at 1 in., then Wellington's formula reduces to Wh.

In regard to the Wellington formula, which the speaker has always
been inclined to favor in the above modified form, it must be under-
stood distinctly that, like all empirical expressions, its use applies
only to a very restricted range of variation, such as is ordinarily met
in practical pile-driving. That is to say, it is not to be taken as a

Mr. Gould. general proposition in applied mechanics. If we so regard it, we shall be involved in the practical absurdity of admitting the effect upon the pile to be the same whether a 50-lb. hammer and 1 000 ft. fall were used, or a 5 000 000-lb. hammer and 0.01 ft. fall.

If specifications called for a hammer weighing 2 000 lbs. and a fall of 20 ft., giving 40 000 foot-pounds, and it were found that only a 1 500-lb. hammer were available, it might, under the circumstances, be admissible to use the lighter weight with a 27-ft. drop, but not one of 1 000 lbs. and a 40-ft. drop, the effect of which would be quite different from that intended.

Mr. Haswell uses the square root of the height, instead of the height itself, in his formula. The speaker is inclined to believe that this is correct, as regards the work practically done by the hammer, and with the distinct understanding that the formula applies, like the Wellington, only to comparatively small differences of fall. The formula also contains a factor of safety, varying from 3 to 6. Taking the higher value, and using round numbers, it will be seen that Mr. Haswell's formula reduces to

$$L = 5 \ W \sqrt{h,}$$

in which $L =$ the safe ultimate load, expressed in the same unit as the weight of the hammer.

It has already been shown that Wellington's formula, for inch refusal, reduces to

$$L = W \ h.$$

Within the ordinary range of height of fall and admitting a refusal of 1 in. in both, the two will not differ very widely, and the speaker is inclined to believe that Mr. Haswell is correct in minimizing the effect of height, particularly as this tends to encourage the use of heavy hammers and low drops.

The whole subject of pile-driving is one of judgment rather than of mathematics. Mr. Le Conte has mentioned some of the curious changes in the set of a pile brought about by time, according to the character of the ground penetrated. It seems, however, hardly worth while to consider the case of such materials as sand, gravel or hardpan. Generally, when such hard formations are reached, pile-driving is stopped and a case of bearing piles is presented. Soft material under or above water is what we are most concerned with, and it is a matter of common observation that piles driven in such material become so bound by skin friction, that, after 24 hours of repose, their refusal becomes much greater than during the original driving. This seems to be true to a marked degree in the case of jetted piles. In such material, piles should be sent down as rapidly as possible by a continuous volley of quick blows. This would work havoc with our formulas, unless the pile driver were sent back again next day to test the piles for final set; but the practical result would be superior.

Horace J. Howe, M. Am. Soc. C. E.—The results of the experiments Mr. Howe.
in Brooklyn by the late W. J. McAlpine, Past-President, Am. Soc. C. E.,
have been published in many papers. One of his conclusions is, that
the effect of increasing the fall of the ram is to increase the sustaining
power of the pile in the ratio of the square root of the fall.

Another conclusion is, that by adding to the weight of the ram the
sustaining power of the pile was increased by $\frac{7}{10}$ to $\frac{9}{10}$ of the amount
used to the ratio of the augmented weight of the ram.

As a result of Mr. Haswell's experiments with 1-lb. weights, he
presents a formula for the safe bearing power of a pile, in terms of the
weight and fall of the hammer, an assumed penetration of $\frac{1}{2}$ in. or less
(which does not appear separately) and a factor of safety. As in other
instances, this safe load is found varying directly with W, and a
function of h and inversely as F; and there seems to be no new term
added to make the formula distinctive.

Passing from the question as to whether the experiments have been
interpreted properly, to the general question of the application of a
formula to common practice, there seems to be the necessity of defin-
ing, as far as possible, the limit of the reliability of any formula.

All that can be claimed for any formula for the safe load on a pile
is governed by the condition of the ground at the time of driving.

If we follow the career of a pile after the formula has left it, we
enter into the domain of history, and it is a question of how far into
history we care to go. If we wish short-time tests, say a week, or a
year, we may arrive at certain conclusions. If we wish long-time
tests, say 40 or 50 years, we may arrive at certain other conclusions, or
our successors may. But a formula is only a neat expression for a
very short-time test indeed, and the emphasis, at the present time,
should be placed on the deleterious action of long-time, and more
or less continuous, forces.

For instance, take the movement of a railroad train near a building.
The speaker has sat in a frame house 1 000 ft. from the track, and has
been astonished many a time at the noise made by the structure, due
to the vibration transferred through dry loamy soil to the masonry
foundations, and thence up to the top story.

Again, another example is a factory on Fort Point Channel, Boston,
in which the movements of the piles are seen plainly when the machinery
is in motion. A long pendulum is set up inside the building to chron-
icle the changes. The building is new, and the blue clay is pretty
reliable, and, therefore, the settlement, so far, has been slight.

Sometimes we see or hear of a layer of concrete 5 or 6 ft. thick
going into a foundation, without, perhaps, being able to find the
man who figured the exact thickness required. In the light of these
principles, and with a pile foundation, it may not be altogether a waste
of money.

Mr. Howe.   Then, water is another long-time agent, tending not only to convey lubricants to the pile, but to move the adjacent soil perhaps laterally.

The matter of incident decay, too, is of the widest interest. It involves a knowledge of the proper elevation above low water at which to cut off the pile or to limit the grillage in different soils and for different purposes. The determination of this limit is still in the experimental stage, and depending on the years as an umpire of opinions.

Then another point, beyond the reach of the formula, is the relative supporting power of one pile, and of a mass of piles; or the relative supporting power of a small and of a large surface. This is also in the experimental stage.

Meanwhile, the pile, with its never-sleeping load, takes advantage of any errors of judgment on the part of the engineer, and slips down and down and never rights itself. So that, for work which is to be permanent, the emphasis should be on the long-time experiments.

Experiments to the ultimate, of any kind, are rare, and it may be the proper time to say that it is scarcely creditable to the profession. Recently, in a room full of engineers, the question was asked as to ultimate experiments, and there was no answer.

The profession is indebted to Mr. Haswell for again bringing this condition of things to its attention.

Mr. Le Conte.   L. J. LE CONTE, M. Am. Soc. C. E. (by letter).—The writer is much interested in this subject, and is pleased with the manner in which it is presented. The author is correct in his criticism of existing formulas. The true cause for this extraordinary discrepancy in computed results, as determined by standard formulas, is to be traced to the fundamental fallacy of assuming that the resistance of the soil to the penetration of the pile is constant, for any given depth, during the time the pile is being driven, and that this condition remains constant after the pile is driven. Both of these assumptions are absolutely untenable. Hence the writer has considered the problem as being largely beyond the reach of mathematical formulas which demand rigid facts as a foundation, and this rigidity does not exist. The following notes, based on personal experience, will go to show the absurdity of relying upon the computed results as obtained from the use of the ordinary standard formulas.

Case 1. Sandy Soils.—Soon after driving begins, a cone of resistance forms at and near the foot of the pile, and the material comprising this cone becomes more and more compressed as the pile descends, the penetration at each blow becoming less and less until it reaches a final minimum, of, say, 0.5 in., when the driving is completed. Now, wait 24 hours or more, and, by way of experiment, begin driving a second time. It will be observed that the same pile now penetrates 4 to 5 ins. at the first blow. What does this mean? It simply means that the compressed material surrounding the pile has had time to dissi-

pate, or readjust itself, so that the soil has resumed its normal condition, Mr. Le Conte. such as existed before any driving was done.   It would appear, therefore, that, in sandy soil, the penetration at the last blow is a very dangerous and misleading element to figure on, for the reason that it is dependent upon a temporary, abnormally compressed condition of the soil, which is always short-lived, and cannot be depended upon as a reliable function in any formula.   It is evident that, in sandy soil, the Dutch formula, by *pro rata* of number of blows, is, for obvious reasons, preferable to the final-penetration formulas.

Case 2.   Plastic Muds and Clays.—The physical conditions are entirely different from those in Case 1.   It is true that the penetration at each successive blow becomes less and less until the driving is completed, that at the last blow being, say, 3 ins.   Now, as before, wait 24 hours or more and begin driving the same pile a second time.   It will be found that the hammer, after repeated blows, cannot start the pile at all.   In this case, then, the penetration at the last blow, 3 ins., is no index whatever as to what load the pile will withstand.   The blow necessary to start the pile after standing at rest for 24 to 48 hours is the better test.   This curious physical property of the plastic muds and clays has been properly likened to the similar results obtained by forcing a pin into a solid india-rubber ball.   The same force which pushed it in will pull it out again if it is pulled out immediately; but, after waiting 24 hours, the force required will be five times as great.

Case 3.   Hardpan and Sandy Clays.—In this material the most perplexing results of all are to be found.   The writer has never seen any two hardpans which developed the same results.   The ultimate result depends, not only upon the percentage of clay and sand in the hardpan, but also upon the solubility of the clay when brought into contact with the local ground-water.   Where piles are driven through ground-water overlying hardpan, the water invariably follows each pile down as it penetrates the ground, and softens the clay in contact with the surface of the pile, and often practically destroys all lateral friction.   In such cases the only point of support is at the end of the pile, and the overhead load will be supported by a cluster of columns (piles), the heights of which are equal to the length of the piles.

Finally, there is no doubt about the fact that the best way to test the bearing power of piles, in any given ground, is to drive experimental piles to the required depth, then load them with pig-iron and observe the effects after long standing, and thus find by trial their ultimate resistance.   This is not always practicable, in which case the next best way is to drive experimental piles to the depth required or specified, then let them rest for 48 hours or more and drive them a second time, and note the drop of the hammer necessary to start the pile and the first penetration in inches.   The results thus obtained represent more nearly the resistance of the soil in its normal condition.

Mr. Haswell.   CHARLES H. HASWELL, M. Am. Soc. C. E. (by letter).—In response
to the remarks of Mr. Crowell, on the results and deductions of the
author's experiments to ascertain the relative impact of falling weights
from different heights, it was not essayed, nor was it held to be at all
practicable, to determine exact measures of impact with an instrument
possessing such retarding elements as a spring-balance, supplemented
with a detent.   It was essayed only to ascertain the relation and ratio
of the impact of the weights from different heights of fall, and as like
causes produce like effects, the relation of the results, satisfactorily
determined by repeated operations, were just as conclusive as if
obtained with the exactness of the requirements of normal results.

Regarding the final penetration of 0.5 in. being assumed as the
base of the formula submitted, it was taken, inasmuch as it is not only
the ordinary limit of practice, but it is a sufficient refusal and the
practically proper one; as the difference between it and a less depres-
sion is not only difficult to observe, but the attainment of it is of little
relative value with the usual factor of safety, and when effected, it
results frequently in the splitting of the piles; further, if it is differ-
ent, the factor can be altered to meet the conditions, and in any for-
mula, if the final penetration is not a fixed element, computation of
the capacity of a pile is emasculated, and the judgment of the driver
from visual observation is the only result whereby to assign its
capacity.

A mean factor of 4.5 is assumed, and when it is considered that the
resistance of a pile in one hour after being driven, is, by the subsi-
dence of the soil around it, very much increased, it is held to be a full
measure of safety.   In moist sand, with a rough pile of oak or pine
(not spruce), a factor for a quiescent load is, in the author's opinion,
practically unnecessary, and this opinion is based upon exceptional
opportunities to observe the effect of such conditions.

It is not the author's purpose to discuss the application of *vis viva*
in this case, it is preferred to give observed results.   The resistance
an instrument opposes to a falling weight is assumed as a measure of
its force, whether it is a spring-balance or a supporting pile.   .

In further support of his remarks, Mr. Crowell submits a diagram
illustrative " of the characteristic plotting of the respective formulas "
of Wellington and the one submitted, and that " it will be seen that
for a fall of only 2 ft., the indicated safe load is one-third as great as
that indicated for a fall of 18 ft., nine times as great."   Now, as the
formula gives but three times for the 2 ft., and as $\sqrt{2\ g\ 18}$, is just
three times that for 18 ft., the formula agrees with the illustration,
and the result is not nine times, as asserted.

Mr. Gould is inclined to believe that the use of the $\sqrt{H}$ is correct.
His opinion, from his capacity and great experience, is entitled to
much consideration.

Mr. Howe recites that by the published experiments of Mr. McAl- Mr. Haswell. pine, the sustaining power of a pile is increased in the ratio of the square root of the fall of the ram. This also is in support of the formula used, and is wholly at variance with the formula of Mr. Wellington.

Mr. Le Conte's deduction and experience, regarding the second driving of a pile after an interval of a day in sandy soil, is at variance with the author's experience, and as to the value of the Dutch formula, the author fails to recognize the utility of considering the number of blows of a ram, when the effect of the final blow or impact of it is the measure of the resistance of the pile. The tenacity of a nail in wood is greater when driven at one blow than by a number of them, for with one blow there is but one resilience of the disturbed fibers of the wood, which resistance is the element of the tenacity of the nail, and a repetition of blows is a repetition of the resilience and consequently a weakening of it.

"Molesworth on Pile-Driving" gives a table of the force of a blow of a falling weight $= V\,W$.